Roll No. ..... Total Printed Pages -12

F - 523

M.A./M.Sc. (Second Semester)

EXAMINATION, May - June, 2022

MATHEMATICS

Paper Fifth

[Advanced Discrete Mathematics (II)]

Time : Three Hours] *[Maximum Marks:80*

Note: Attempt all sections as directed.

(Section - A)

(Objective/Multiple Choice Questions)

(1 mark each)

Note: Attempt all questions.

Choose the correct answer:

P.T.O.

1. A graph is G is:

   (A) (V,V)

   (B) (E,E)

   (C) (V,E)

   (D) None of these

2. A vertex of degree one is called:

   (A) Pendent vertex

   (B) Isolated vertex

   (C) Odd vertex

   (D) None of these

3. Finite alternating sequence of vertices and edges is said to:

   (A) Path

   (B) Walk

   (C) Circuit

   (D) None of these

4. Euler formulas for connected planar graph is:

   (A) $v-e-r=2$

   (B) $v-e+r=2$

   (C) $v+e+r=2$

   (D) None of the above

5. A..... is defined as a tree in which there is exactly one vertex of degree two, and each of the remaining vertices is of degree one or three

   (A) Rooted tree

   (B) Binary tree

   (C) Spanning tree

   (D) None of these

6. A connected graph without any circuits is called:

   (A) Binary tree

   (B) Tree

   (C) Spanning tree

   (D) None of these

7. The sum of the path lengths from the root to all pendant vertices; is called

   (A) Path length of the Rooted Tree

   (B) Path length of the Spanning Tree

   (C) Path length of the Binary Tree

   (D) None of these

8. A Tree T is said to be a spanning tree of a connected graph G if

   (A) T contains all edges of G

   (B) T contains all edges & vertices of G

   (C) T contains all vertices of G

   (D) None of these

9. In a connected graph G, a set of edges is:

   (A) Regular set

   (B) Cut - set

   (C) Cut - vertices

   (D) None of these

10. rank + nullity = ...

    (A) Number of vertices in G

    (B) Number of chords in G

    (C) Number of edges in G

    (D) None of these

11. An Isolated vertex in directed graph is a vertex in which

    (A) The in-degree and the out-degree are both equal to zero

    (B) The in-degree and the out-degree are both equal to one

(C) The in-degree one and the out-degree zero

(D) None of these

12. In a directed graph; A vertex with $\deg(v) = 0$ is called:

(A) Sink

(B) Trial

(C) Source

(D) None of the above

13. In computer science the term "Automaton" means:

(A) Indiscrete automaton

(B) Discrete automaton

(C) Automatic automaton

(D) None of these

14. Transition system is:

(A) A finite directed labelled graph

(B) A infinite directed labelled graph

(C) A infinite undirected labelled graph

(D) None of the above

15. For a Moore machine if the input string is of length n, the output string is of length:

(A) n

(B) n + 1

(C) n + 2

(D) None of the above

16. For a Mealy machine, if the input string is of length n, the output string is:

(A) n

(B) n - 1

(C) n + 1

(D) None of the above

17. In a Non deterministic finite automaton (NDFA); δ is the transition function mapping from $Q \times \Sigma$ into.....which is the power set of Q,

(A) 2^Q

(B) 3^Q

(C) 1^Q

(D) None of the above

18. In a Turing machine $b \in \Gamma$ is

    (A) Initial

    (B) Finite

    (C) Blank

    (D) None of the above

19. The Moore machine; δ is the transition function:

    (A) $\Sigma \times Q$ into Σ

    (B) $\Sigma \times Q$ into Q

    (C) $\Sigma \times Q$ into Δ

    (D) None of the above

20. The Turing machine can be thought of as a finite - state automaton connected to a:

    (A) R/R (read/read)

    (B) W/W (write/write)

    (C) R/W (read/write)

    (D) None of the above

Section - B

(Very Short Answer Type Questions)

(2 marks each)

Note: Attempt all questions 2 - 3 sentences.

1. Define simple graph with an example.
2. Define planar graph.
3. Define fundamental cut-set.
4. Define Rooted Tree.
5. Define Homomorphism.
6. Write the statement of Pumping Lemma.
7. Define partial recursive functions.
8. Define Moore machine.

Section - C

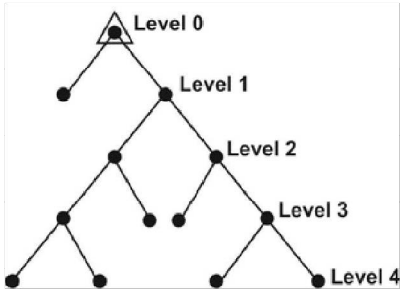
(Short Answer Type Questions)

(3 marks each)

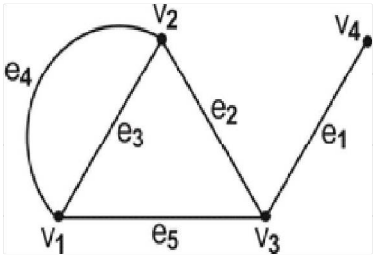
Note: Attempt all questions in less than 75 words.

1. Show that the number of vertices of odd degree in a graph is always even.

2. Find the path length of the Binary tree in the following fig



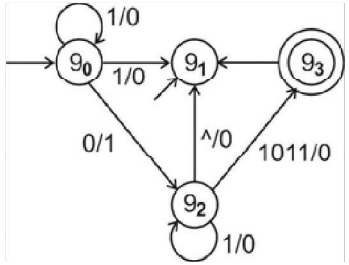
3. Find the incidence matrix for the graph given below-



4. Prove that a tree with n vertices has n - 1 edges.

5. Write short notes on Turing machine & partial recursive functions.

6. Consider the transition system in the following Fig. Determine the initial states, final states and the acceptability of 101011, 111010.



7. Write short notes on Fundamental cut - set.

8. Define Reduced machine with an example.

Section - D

(Long Answer Type Questions)

(5 marks each)

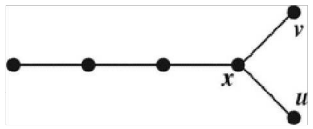
Note: Attempt all questions.

1. Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

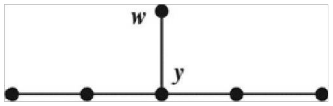
OR

Verify that the following two graphs are isomorphic.

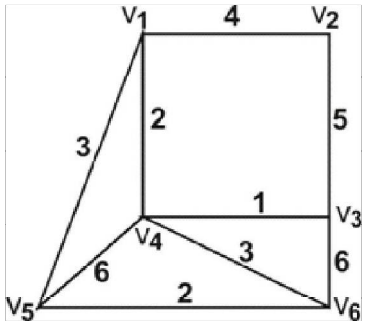
(i)



(ii)

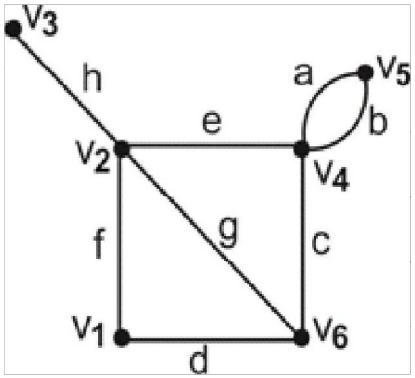


2. Find the minimal spanning tree for the graph in the following figure using Kruskal's algorithm.

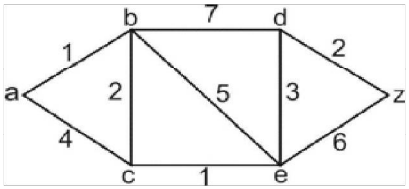


OR

Find the circuit matrix in given graph.



3. Find the shortest path from a to z in the following graph using Dijkstra's algorithm:



OR

In a directed graph with e edges, sum of the in - degree = sum of the out-degree = e, In other words, show that:

$$\sum_{i=1}^{n} d^{-}(v_i) = \sum_{i=1}^{n} d^{+}(v_i) = e$$

4. Construct a mealy machine which is equivalent to the Moore machine given in the following table:

Present State	Next State		Output
	$a = 0$	$a = 1$	
$\rightarrow$ 90	93	91	0
91	91	92	1
92	92	93	0
93	93	90	0

OR

Design a Turning machine to recognise all strings consisting of even number of 1's-.